The Beauty and Joy of Computing

Lecture #7
Algorithms II

UC Berkeley EECS
Sr Lecturer SOE
Dan Garcia

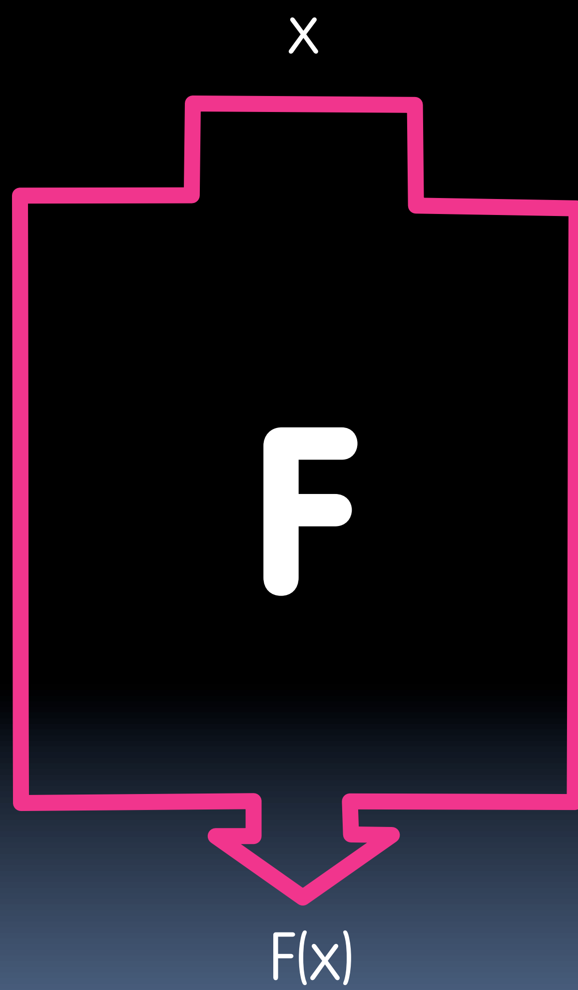
California Law Says All Websites Must Give Minors Option To Delete User Activity

Good: More privacy!
Bad: How exactly do you delete content from the Internet?

http://abcnews.go.com/Technology/calif-law-websites-minors-delete-activity/story?id=20361045/

Functional Abstraction (review)

- A **block**, or **function** has inputs & outputs
  - Possibly no inputs
  - Possibly no outputs (if block is a command)
    - In this case, it would have a "side effect", i.e., what it does (e.g., move a robot)
- The contract describing what that block does is called a **specification** or **spec**



What is IN a spec? (review)

- Typically they all have
  - NAME
  - INPUT(s)
    - (and types, if appropriate)
    - Requirements
  - OUTPUT
    - Can write "none"
  - (SIDE-EFFECTS)
  - EXAMPLE CALLS
- Example
  - NAME : **Double**
  - INPUT : **n** (a number)
  - OUTPUT: **n + n**

What is NOT in a spec?

- **How!**
  - That's the beauty of a functional abstraction; it doesn't say how it will do its job.
- **Example: Double**
  - Could be n * 2
  - Could be n + n
  - Could be n+1 (n times)
    - if n is a positive integer
- **This gives great freedom to author!**
  - You choose Algorithm(s)!

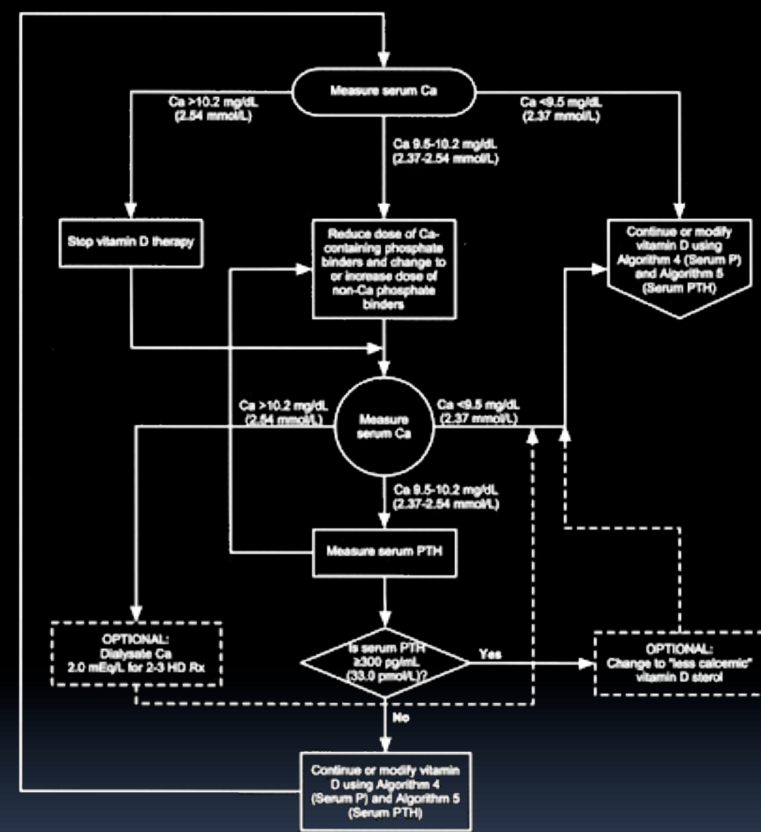
What do YOU think?

Which factor below is the most important in choosing the algorithm to use?

A. Simplest?
B. Easiest to implement?
C. Takes less time?
D. Uses up less space (memory)?
E. Gives a more precise answer?

Algorithm analysis : the basics

- An algorithm is **correct** if, for every input, it reports the correct output <u>and</u> doesn't run forever or cause an error.
  - Incorrect algorithms may run forever, or may crash, or may not return the correct answer.
    - They could still be useful!
    - Consider an approximation…
  - For now, we'll only consider <u>correct</u> algorithms



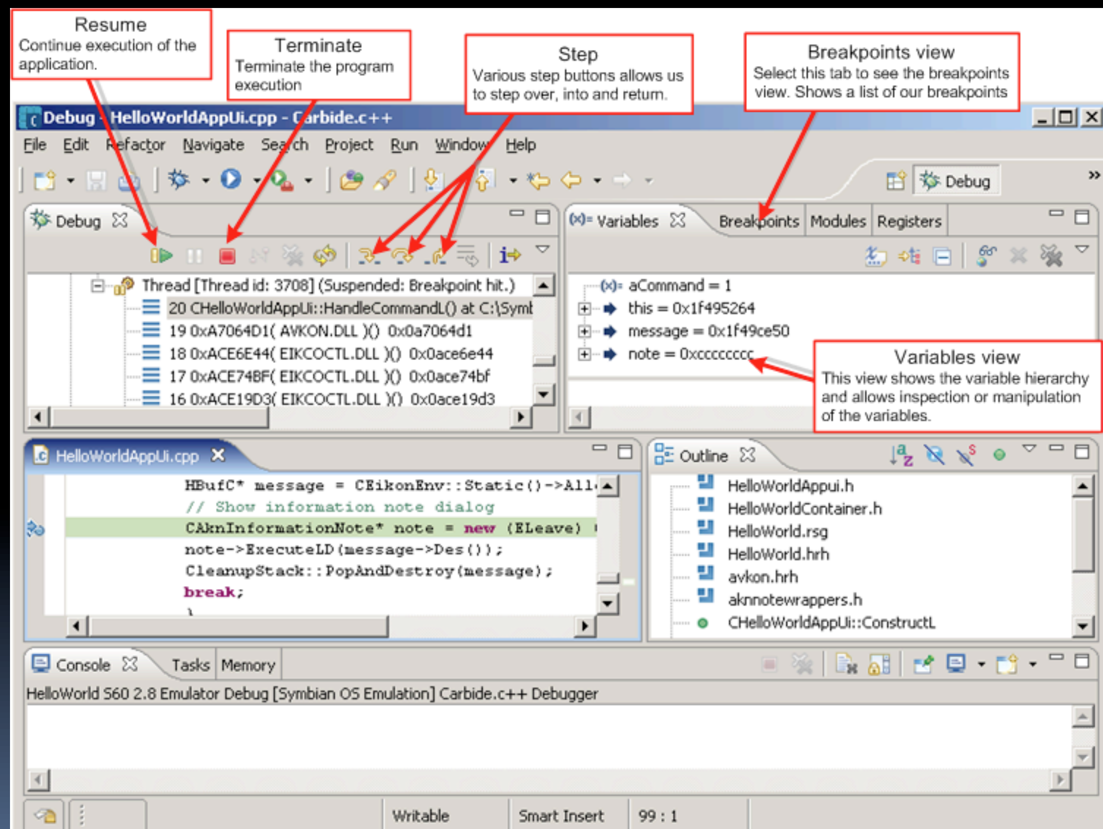
Algorithm for managing Vitamin D sterols based on serum calcium levels.

www.kidney.org/professionals/kdoqi/guidelines_bone/guide8b.htm

Garcia

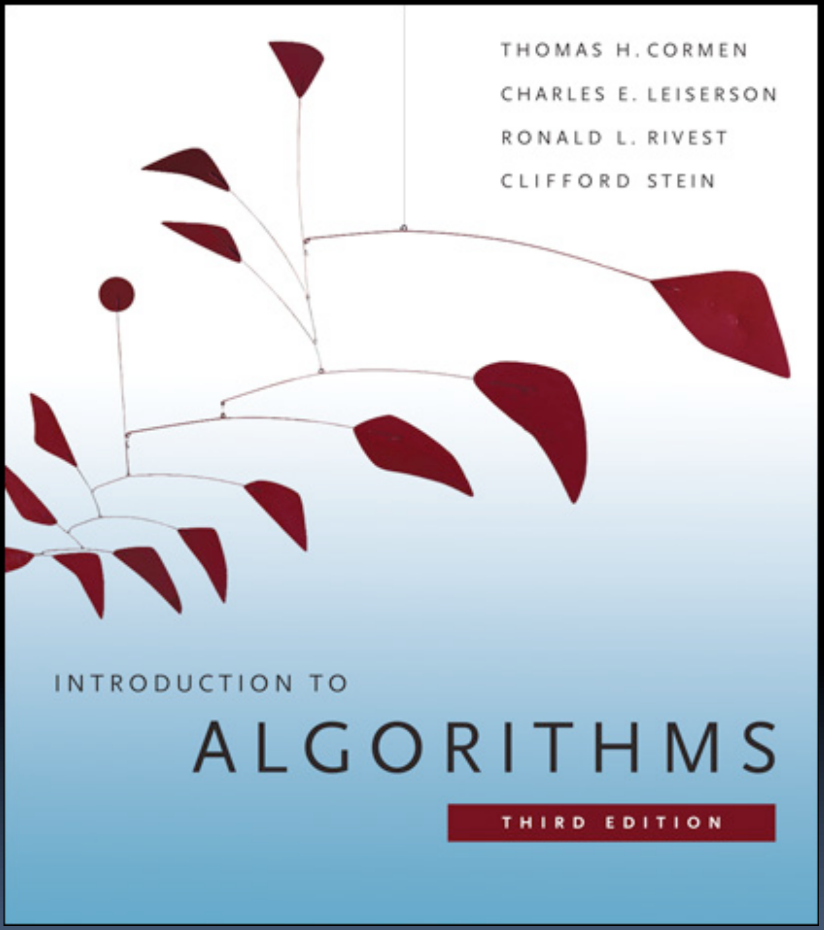
How do you know if "it" is correct?

- Mathematical proof for algorithms
- Empirical verification through testing of programs:
  - Unit Testing
  - Debugging



Reference text

- **This book launched a generation of CS students into Algorithm Analysis**
  - It's on everyone's shelf
  - It might be hard to grok at this point, but if you go on in CS, remember it & own it!
    - …but get the most recent years



Algorithm analysis : running time

- **One commonly used criterion in making a decision is running time**
  - how long does the algorithm take to run and finish its task?
- **How do we measure it?**

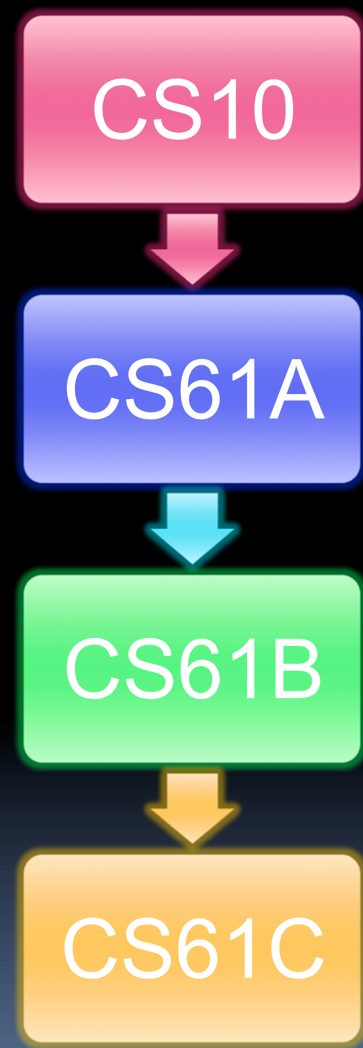
Garcia

Runtime analysis problem & solution

- **Time w/stopwatch, but…**
  - Different computers may have different runtimes. ☹
  - Same computer may have different runtime on the <u>same</u> input. ☹
  - Need to implement the algorithm first to run it. ☹
- ***Solution*: Count the number of "steps" involved, not time!**
  - Each operation = 1 step
  - *If we say "running time", we'll mean # of steps, not time!*

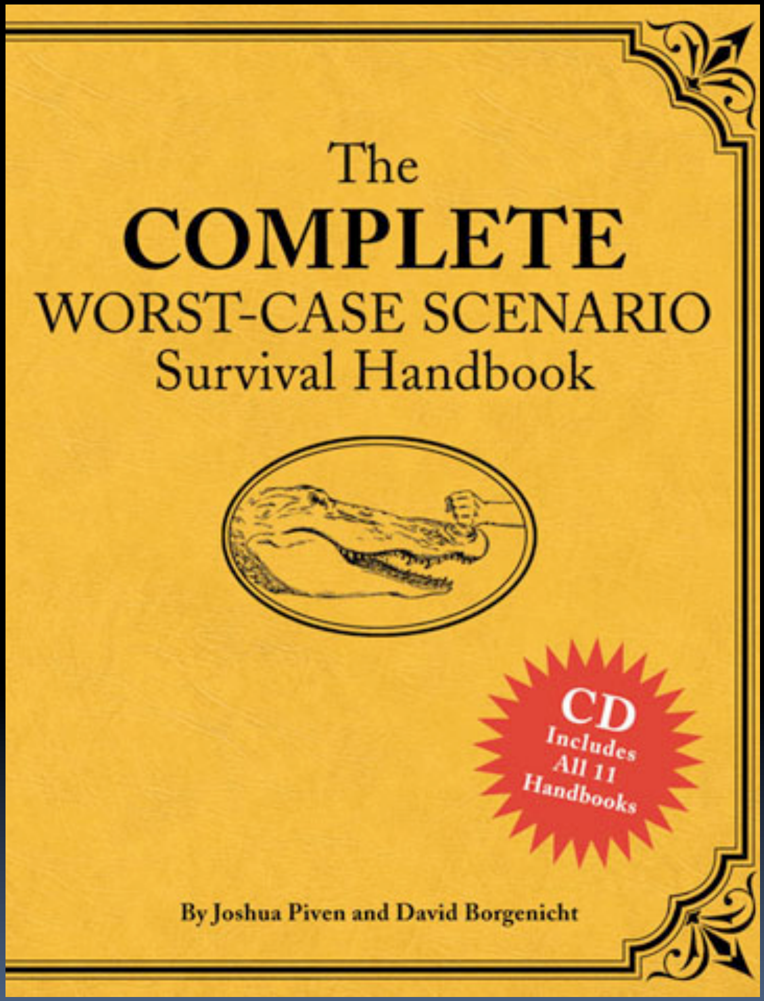
Runtime analysis : input size & efficiency

- **Definition**
  - Input size: the # of things in the input.
  - E.g., # of things in a list
  - Running time as a function of input size
  - Measures efficiency
- **Important!**
  - In CS10 we won't care about the efficiency of your solutions!
  - …in CS61B we will



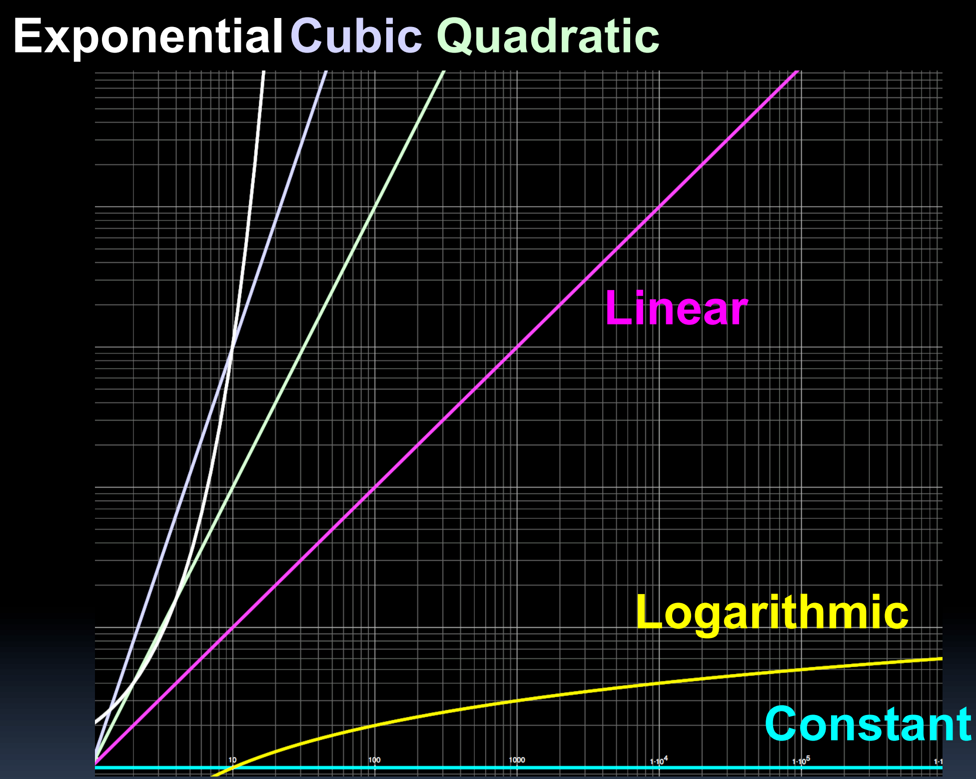
Runtime analysis : worst or avg case?

- **Could use avg case**
  - Average running time over a vast # of inputs
- **Instead: use worst case**
  - Consider running time as input grows
- **Why?**
  - Nice to know most time we'd ever spend
  - Worst case happens often
  - Avg is often ~ worst



Runtime analysis: Final abstraction

- **Instead of an exact number of operations we'll use abstraction**
  - Want order of growth, or dominant term
- **In CS10 we'll consider**
  - Constant
  - Logarithmic
  - Linear
  - Quadratic
  - Cubic
  - Exponential
- **E.g. $10\ n^2 + 4 \log n + n$**
  - …is quadratic



Graph of order of growth curves on log-log plot

Example: Finding a student (by ID)

- **Input**
  - Unsorted list of students L
  - Particular student S
- **Output**
  - True if S is in L, else False
- **Pseudocode Algorithm**
  - Go through one by one, checking for match.
  - If match, true
  - If exhausted L and didn't find S, false

- **Worst-case running time as function of the size of L?**
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

Garcia

Example: Finding a student (by ID)

- **Input**
  - Sorted list of students L
  - Particular student S
- **Output : same**
- **Pseudocode Algorithm**
  - Start in middle
  - If match, report true
  - If exhausted, throw away half of L and check again in the middle of remaining part of L
  - If nobody left, report false

- **Worst-case running time as function of the size of L?**

1. Constant
2. Logarithmic
3. Linear
4. Quadratic
5. Exponential

Example: Finding a student (by ID)

- **What if L were given to you in advance and you had infinite storage?**
  - Could you do any better than logarithmic?

- **Worst-case running time as function of the size of L?**

1. Constant
2. Logarithmic
3. Linear
4. Quadratic
5. Exponential

Example: Finding a shared birthday

- **Input**
  - Unsorted list L (of size n) of birthdays of team
- **Output**
  - True if any two people shared birthday, else False
- **What's the worst-case running time?**

- **Worst-case running time as function of the size of L?**
  1. Constant
  2. Logarithmic
  3. Linear
  4. Quadratic
  5. Exponential

Example: Finding subsets

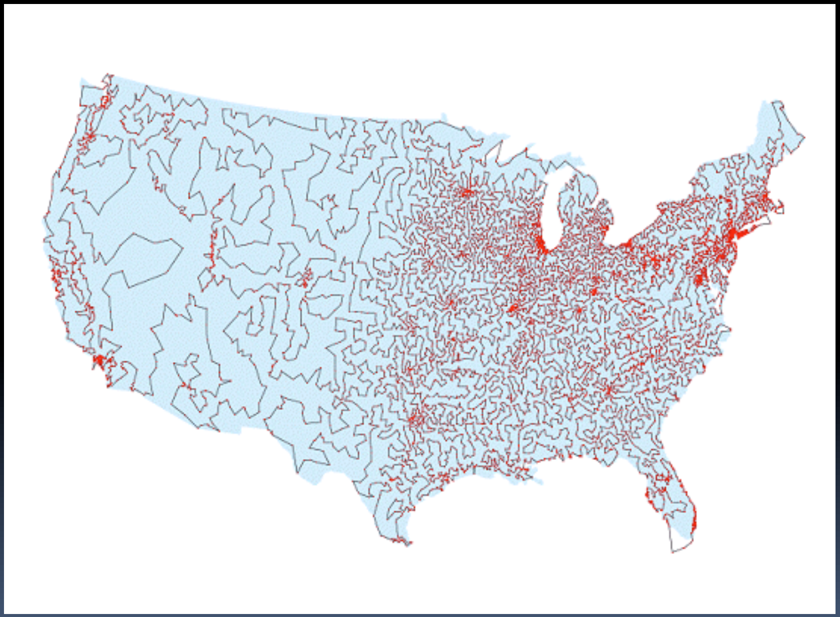
- **Input:**
  - Unsorted list L (of size n) of people
- **Output**
  - All the subsets
- **Worst-case running time? (as function of n)**
- **E.g., for 3 people (a,b,c):**
  - 1 empty: { }
  - 3 1-person: {a, b, c}
  - 3 2-person: {ab, bc, ac}
  - 1 3-person: {abc}

- **Worst-case running time as function of the size of L?**

1. Constant
2. Logarithmic
3. Linear
4. Quadratic
5. Exponential

Limits

- **We can prove mathematically that some algorithms are never solveable!**
- **We can (almost) prove mathematically that some algorithms will never be efficient!**
  - Famous problem P = NP ?
  - Example:
    Travelling Salesman Problem
  - BUT: Can use heuristics for approximation



Summary

- When developing an algorithm, could optimize for
  - Simplest
  - Easiest to implement?
  - Most efficient
  - Uses up least resources
  - Gives most precision
  - …
- In CS10 we'll consider
  - Constant
  - Logarithmic
  - Linear
  - Quadratic
  - Cubic
  - Exponential
- There are empirical and formal methods to verify efficienct and correctness
- Some algorithms cannot be implemtented efficiently

Garcia